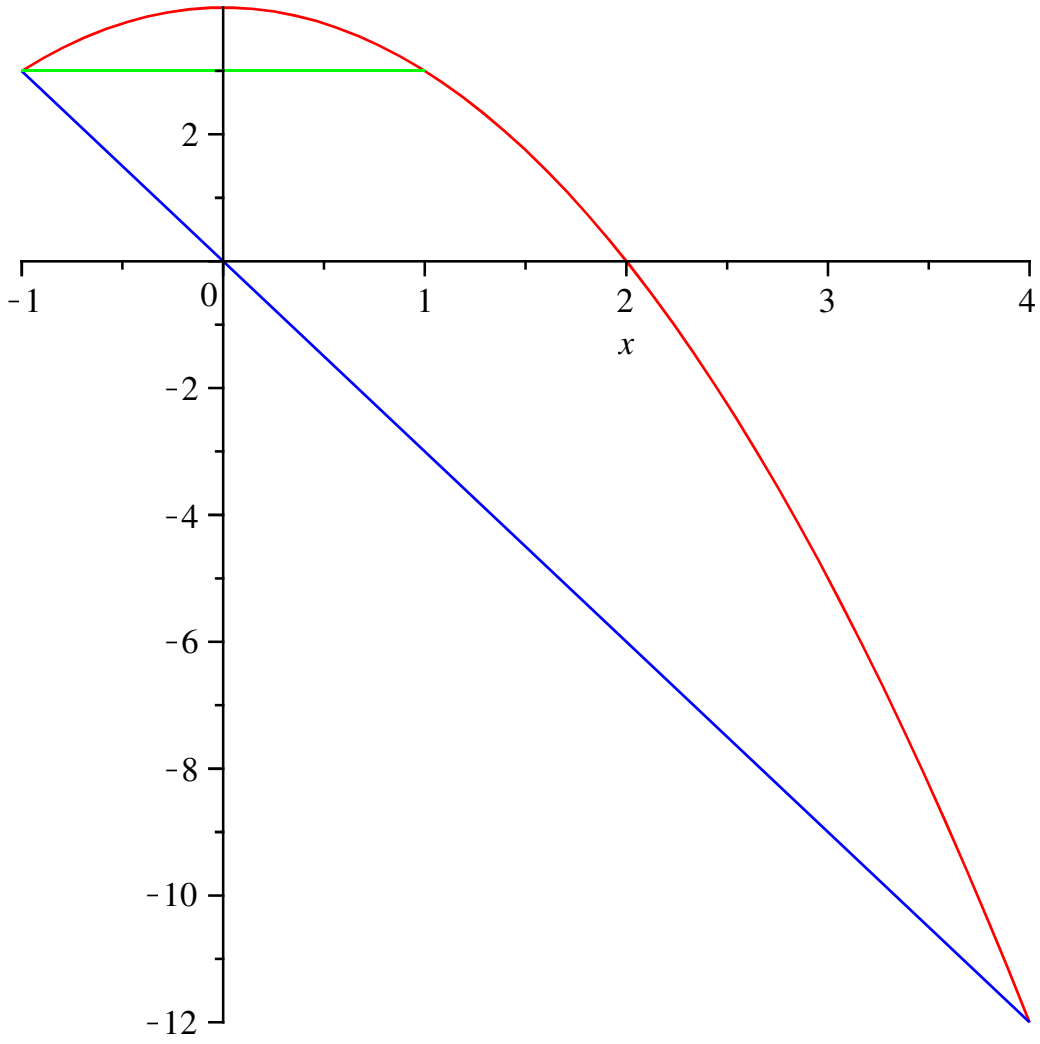
```
> restart:with(plots):with(student):
```

ex1

```
> F:=plot(4-x^2,x=-1..4,color=red):G:=plot(-3*x,x=-1..4,color =
  blue):H:=plot(3,x=-1..1,color = green):
```

```
> display(F,G,H);
```



horizontal strips

```
> x1:=sqrt(4-y); x2:= -y/3;
```

$$x1 := \sqrt{4 - y}$$

$$x2 := -\frac{1}{3}\, y \qquad \textbf{(1)}$$

```
> Int( x1-x2,y= -12 .. 3) + 2* Int(x1,y=3..4)=int( x1-x2,y= -12 ..
  3) + 2* int(x1,y=3..4);
```

(2)

$$\int_{-12}^{3} \left(\sqrt{4-y} + \frac{1}{3}\, y \right) dy + 2 \left(\int_{3}^{4} \sqrt{4-y}\, dy \right) = \frac{125}{6} \quad (2)$$

vertical strips

```
> y1:=(4-x^2); y2:= -3*x;
```

$$y1 := 4 - x^2$$

$$y2 := -3\, x \quad (3)$$

```
> Int( y1 - y2, x = -1 .. 4) = int( y1 - y2, x = -1 .. 4) ;
```

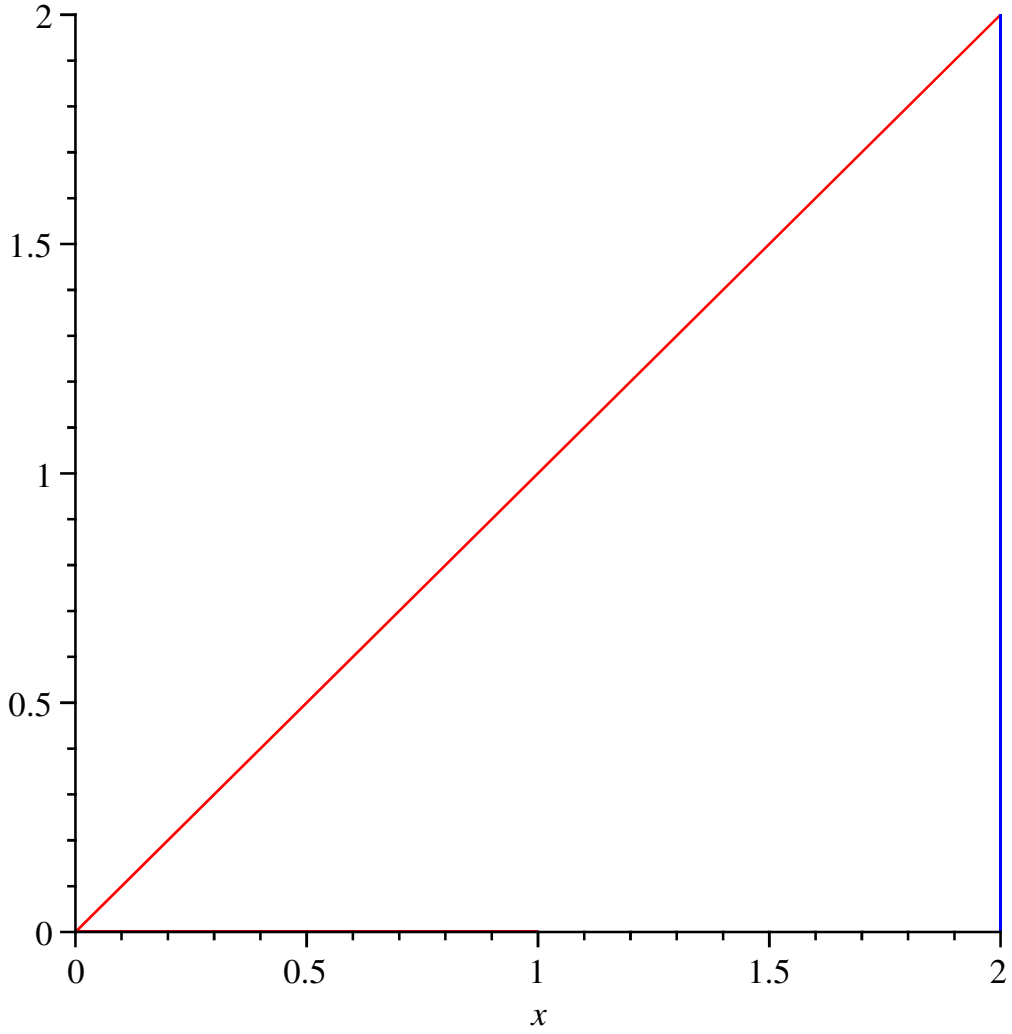
$$\int_{-1}^{4} \left(4 - x^2 + 3\, x\right) dx = \frac{125}{6} \quad (4)$$

```
> restart:with(plots):with(student):
```

ex2

```
> F:=plot(x,x=0..2,color=red):G:=implicitplot(x=2,x=0..4,y=0 .. 2,
  color = blue):H:=plot(0,x=0..1,color = red):
```

```
> display(F,G,H);
```



horizontal strips around y axis is the washer method

```
> Ro := 2; Ri:= x;x:=y;
```

$$Ro := 2$$

$$Ri := x$$

$$x := y \tag{5}$$

```
> Pi*Int( Ro^2-Ri^2,y= 0.. 2);Pi*Int( 2^2-y^2,y= 0.. 2)=Pi*int(
  2^2-y^2,y= 0.. 2);
```

$$\pi \left(\int_0^2 (4 - y^2)\, dy \right)$$

$$\pi \left(\int_0^2 (4 - y^2)\, dy \right) = \frac{16}{3}\, \pi \tag{6}$$

vertical strips around y axis is the shell method

```
> y:=x;h:=y; R:=x;
```

$$y := y$$

$$h := y$$

$$R := y \tag{7}$$

```
> 2*Pi*Int( R*h,x= 0.. 2);2*Pi*Int( x*x,x= 0.. 2)=2*Pi*int( x*x,x=
  0.. 2);
```

$$2\,\pi \left(\int_0^2 y^2 \, dy \right)$$

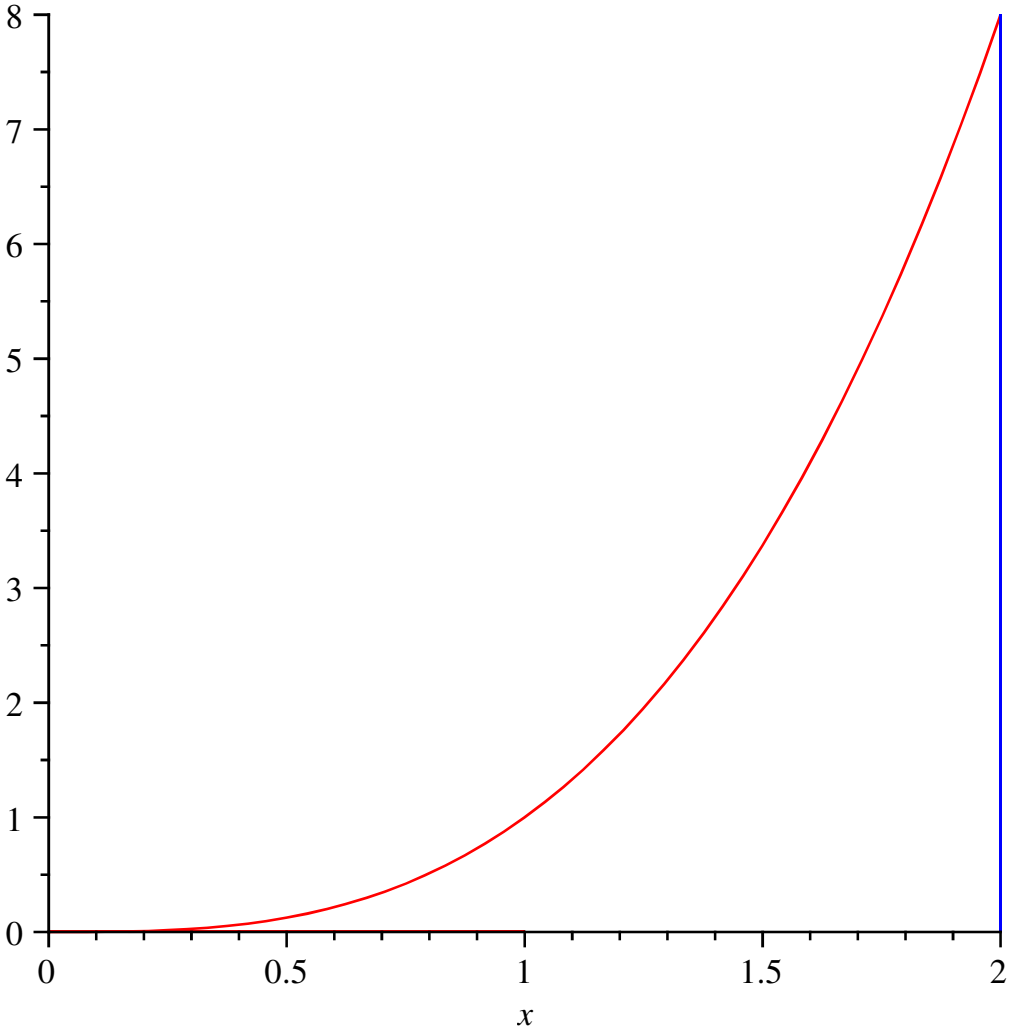
$$2\,\pi \left(\int_0^2 y^2 \, dy \right) = \frac{16}{3}\,\pi \tag{8}$$

```
> restart:with(plots):with(student):
```

ex3

```
> F:=plot(x^3,x=0..2,color=red):G:=implicitplot(x=2,x=0..4,y=0 ..
  8,color = blue):H:=plot(0,x=0..1,color = red):
```

```
> display(F,G,H);
```



horizontal strips around y axis is the washer method

```
> Ro := 2; Ri:= x;x:=y^(1/3);
```

$$Ro := 2$$

$$Ri := x$$

$$x := y^{1/3} \tag{9}$$

```
> Pi*Int( Ro^2-Ri^2,y= 0.. 8);Pi*Int( 2^2-y^(2/3),y= 0.. 8)=Pi*int
  ( 2^2-y^(2/3),y= 0.. 8);
```

$$\pi \left(\int_0^8 \left(4 - y^{2/3}\right) \, dy \right)$$

$$\pi \left(\int_0^8 \left(4 - y^{2/3}\right) \, dy \right) = \frac{64}{5} \pi \tag{10}$$

```
> restart:with(plots):with(student):
```

vertical strips around y axis is the shell method

```
> y:= x^3; h:=y;  R:=x;
```

$$y := x^3$$

$$h := x^3$$

$$R := x \tag{11}$$

```
> 2*Pi*Int( R*h,x= 0.. 2);2*Pi*Int( x^3*x,x= 0.. 2)=2*Pi*int( x^3*
  x,x= 0.. 2);
```

$$2\,\pi \left(\int_0^2 x^4 \, dx \right)$$

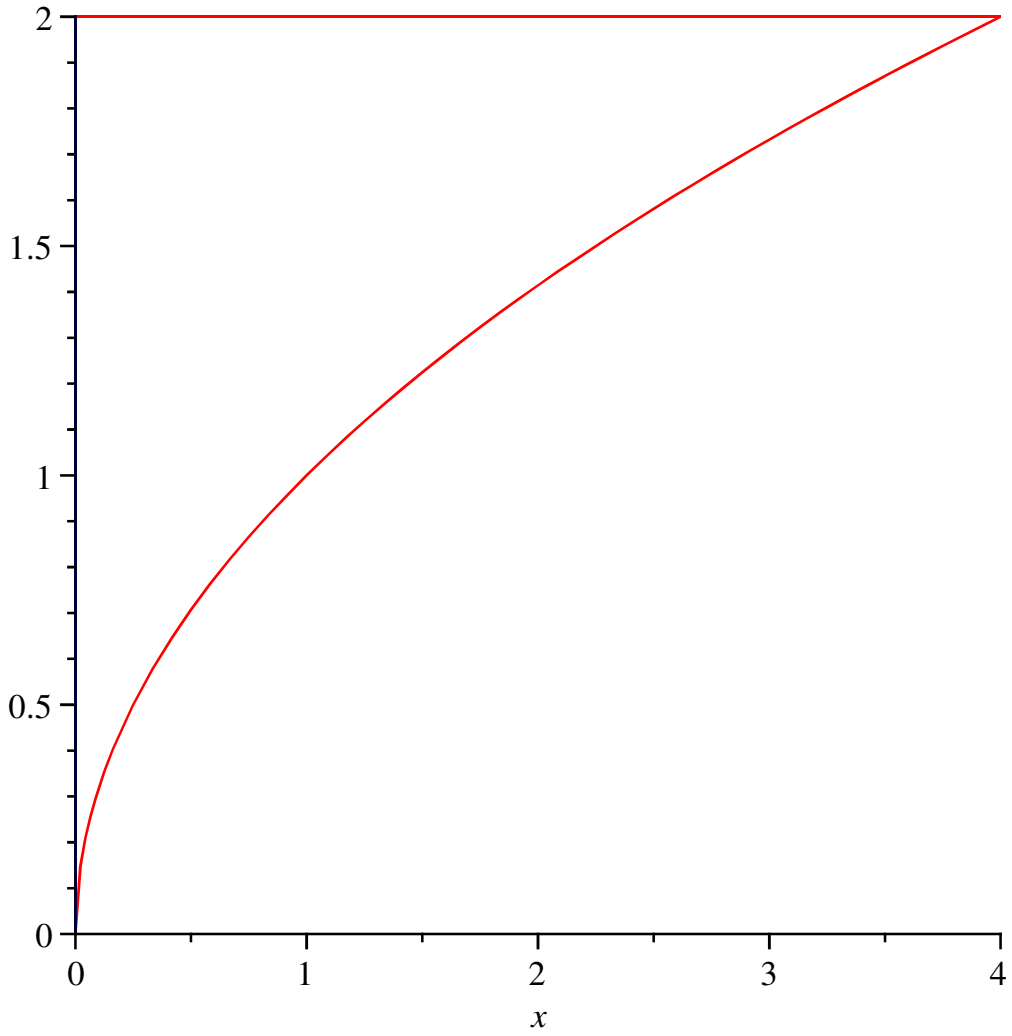
$$2\,\pi \left(\int_0^2 x^4 \, dx \right) = \frac{64}{5}\,\pi \tag{12}$$

```
> restart:with(plots):with(student):
```

ex4

```
> F:=plot(sqrt(x),x=0..4,color=red):G:=implicitplot(x=0,x=0..4,y=0
  .. 2,color = blue):H:=plot(2,x=0..4,color = red):
```

```
> display(F,G,H);
```



vertical strips around x axis is the washer method

```
> Ro := 2; Ri:= y;y:=x^(1/2);
```

$$Ro := 2$$

$$Ri := y$$

$$y := \sqrt{x} \tag{13}$$

```
> Pi*Int( Ro^2-Ri^2,x= 0.. 4);Pi*Int( 2^2-x,x= 0.. 4)=Pi*int( 2^2-
  x,x= 0.. 4);
```

$$\pi \left(\int_0^4 (4 - x)\, dx \right)$$

$$\pi \left(\int_0^4 (4 - x)\, dx \right) = 8\,\pi \tag{14}$$

```
> restart:with(plots):with(student):
```

horizontal strips around x axis is the shell method

```
> h:=x;  R:=y;
```

$$h := x$$

$$R := y \tag{15}$$

```
> 2*Pi*Int( R*h,y= 0..2);2*Pi*Int((y^2)*(y) ,y= 0..2)=2*Pi*int(
  (y^2)*(y),y= 0..2);
```

$$2\,\pi \left(\int_0^2 y\,x\,\mathrm{d}y \right)$$

$$2\,\pi \left(\int_0^2 y^3\,\mathrm{d}y \right) = 8\,\pi \tag{16}$$