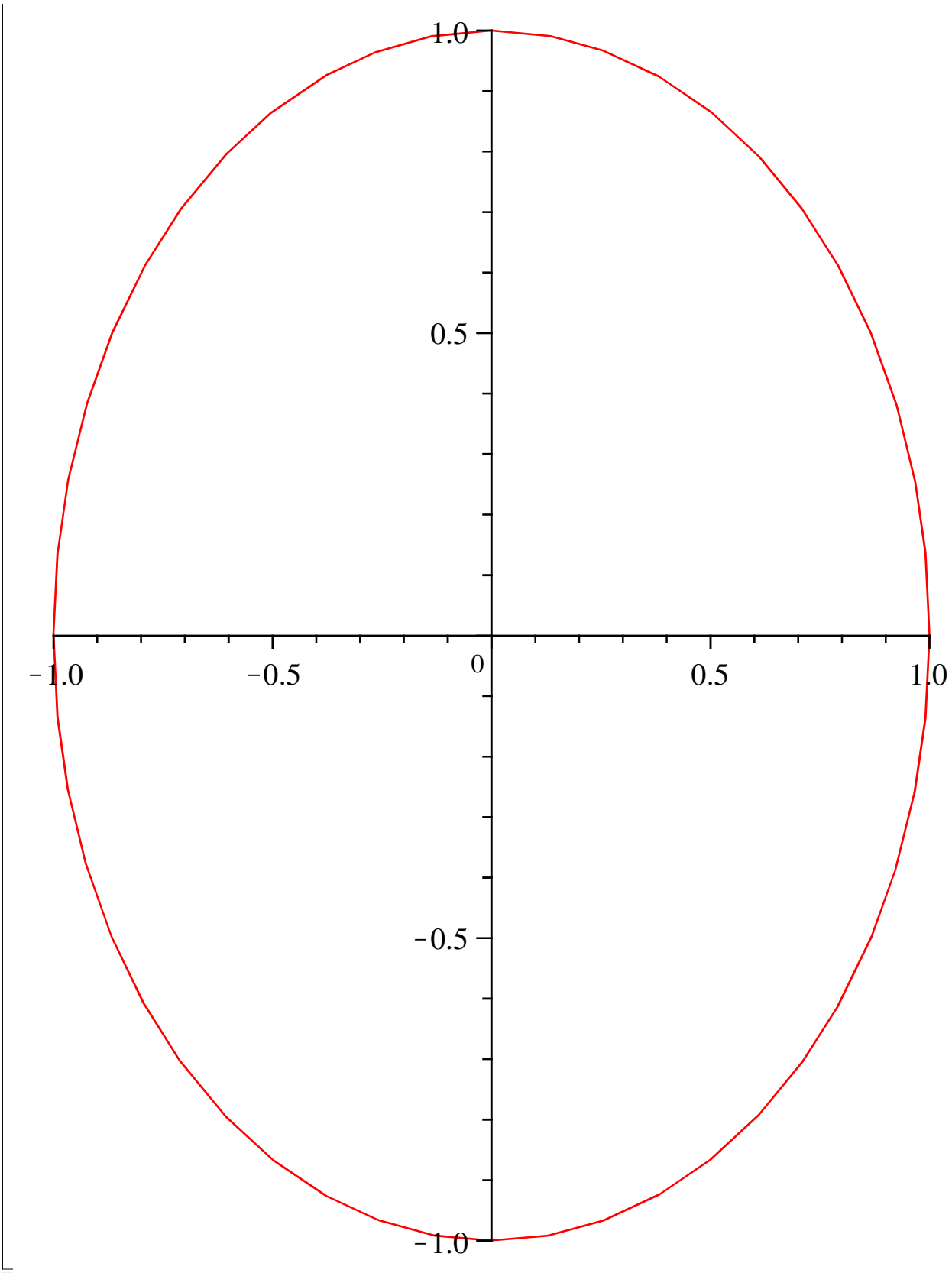
```
> restart:
> with (student):with(plots):
```

Ex1

```
> plot([cos(t),sin(t),t=0..2*Pi]);
```




```
> x:= t -> a*cos(t); y := t -> a*sin(t);
```

$$t \rightarrow a\cos(t)$$
$$t \rightarrow a\sin(t)$$

(1)

```
> Int(sqrt( (Diff(x(t),t))^2 +(Diff(y(t),t)^2)),t=0 .. 2*Pi) = int
  (sqrt( (diff(x(t),t))^2 +(diff(y(t),t)^2)),t=0 .. 2*Pi);
```

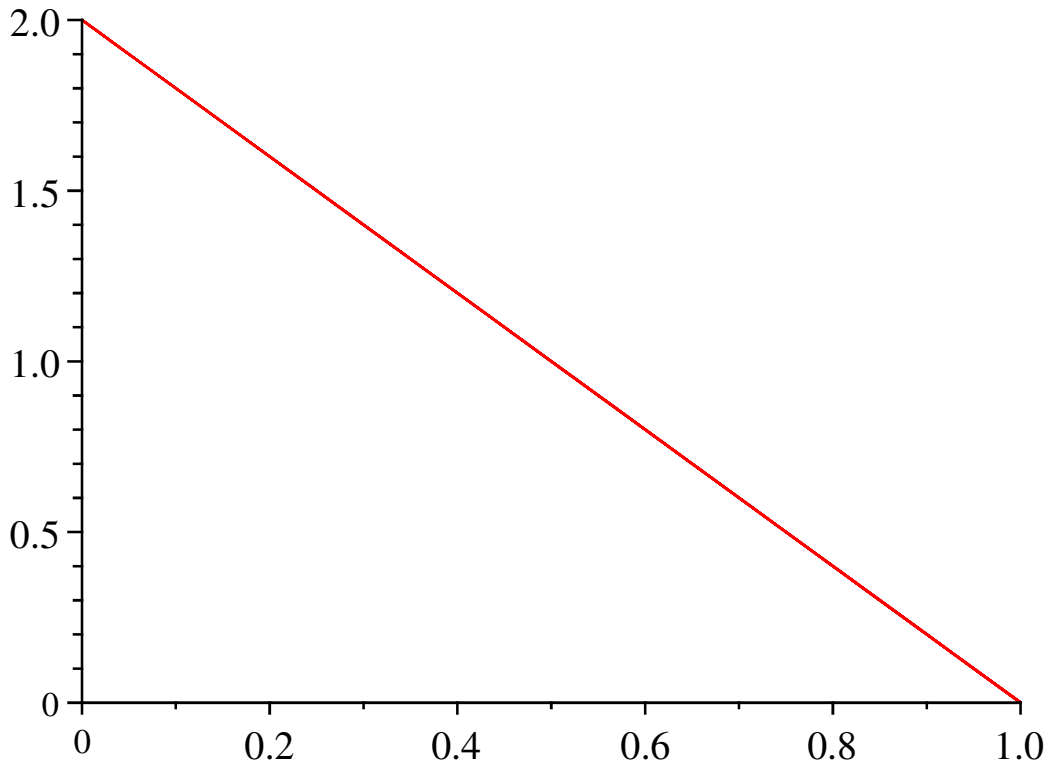
$$\int_0^{2\pi} \sqrt{\left(\frac{\partial}{\partial t}\left(a\cos(t)\right)\right)^2 + \left(\frac{\partial}{\partial t}\left(a\sin(t)\right)\right)^2}\, dt = 2\,\pi\, a\,\mathrm{csgn}(a) \qquad (2)$$

Ex2

```
>
> x:= t -> (sin(t))^2; y := t -> 2*(cos(t))^2;plot([x(t),y(t),t=0.
  .2*Pi]);
> Int(sqrt( (Diff(x(t),t))^2 +(Diff(y(t),t)^2)),t=0 .. Pi/2) = int
  (sqrt( (diff(x(t),t))^2 +(diff(y(t),t)^2)),t=0 .. Pi/2);

>
```

$$t \to \sin(t)^2$$
$$t \to 2\cos(t)^2$$

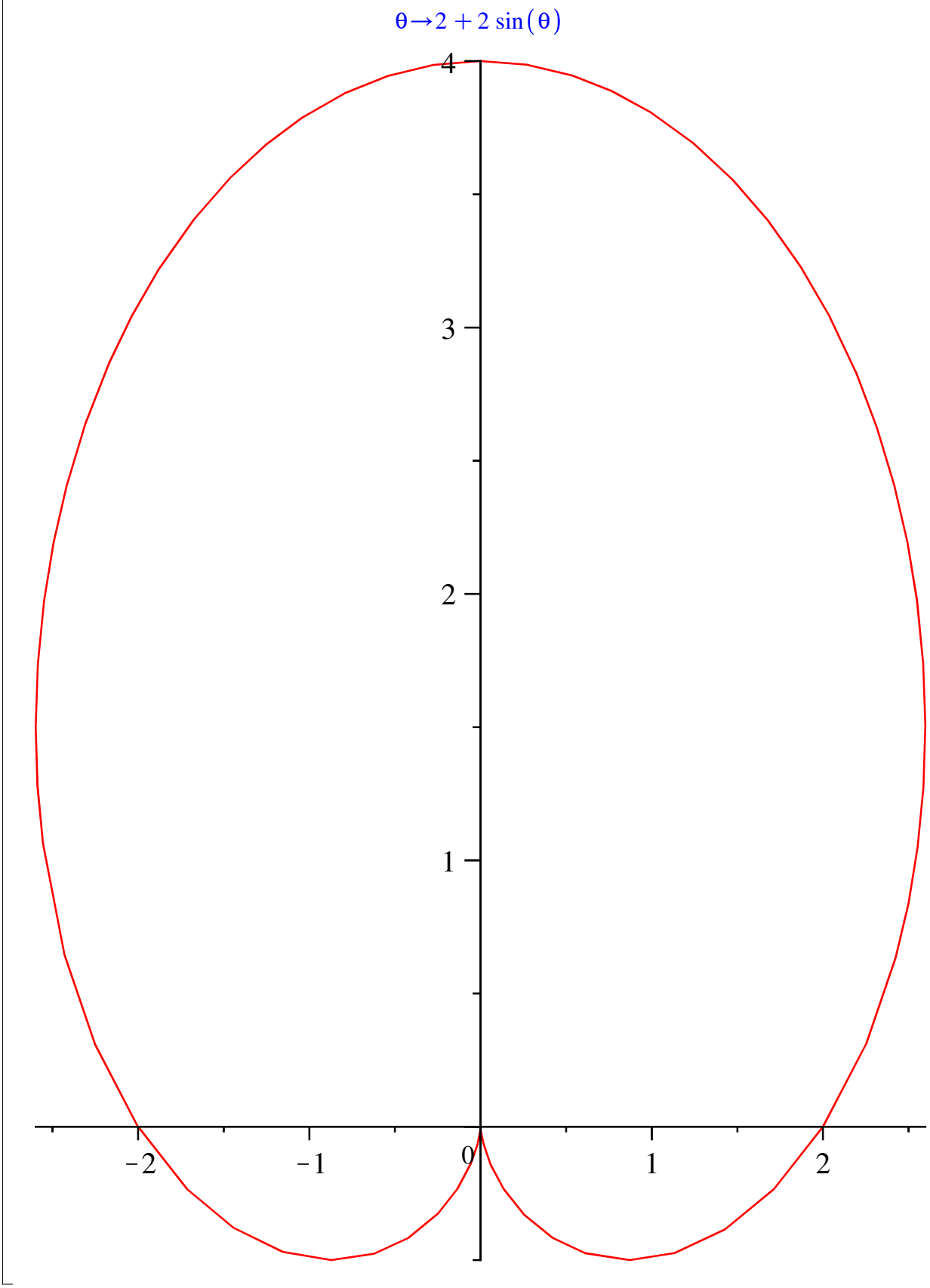


$$\int_0^{\frac{1}{2}\pi} \sqrt{\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\sin(t)^2\right)\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}t}\left(2\cos(t)^2\right)\right)^2}\, dt = \sqrt{5}$$

Ex3

```
> r:= theta  -> 2 + 2*sin( theta);plot([r(theta),theta,theta= 0 .
```

```
.2*Pi],coords=polar);
```



```
> 2*1/2*Int((r(theta))^2,theta= -Pi/2 .. Pi/2)=2*1/2*int((r(theta)
  )^2,theta= -Pi/2 .. Pi/2);
```

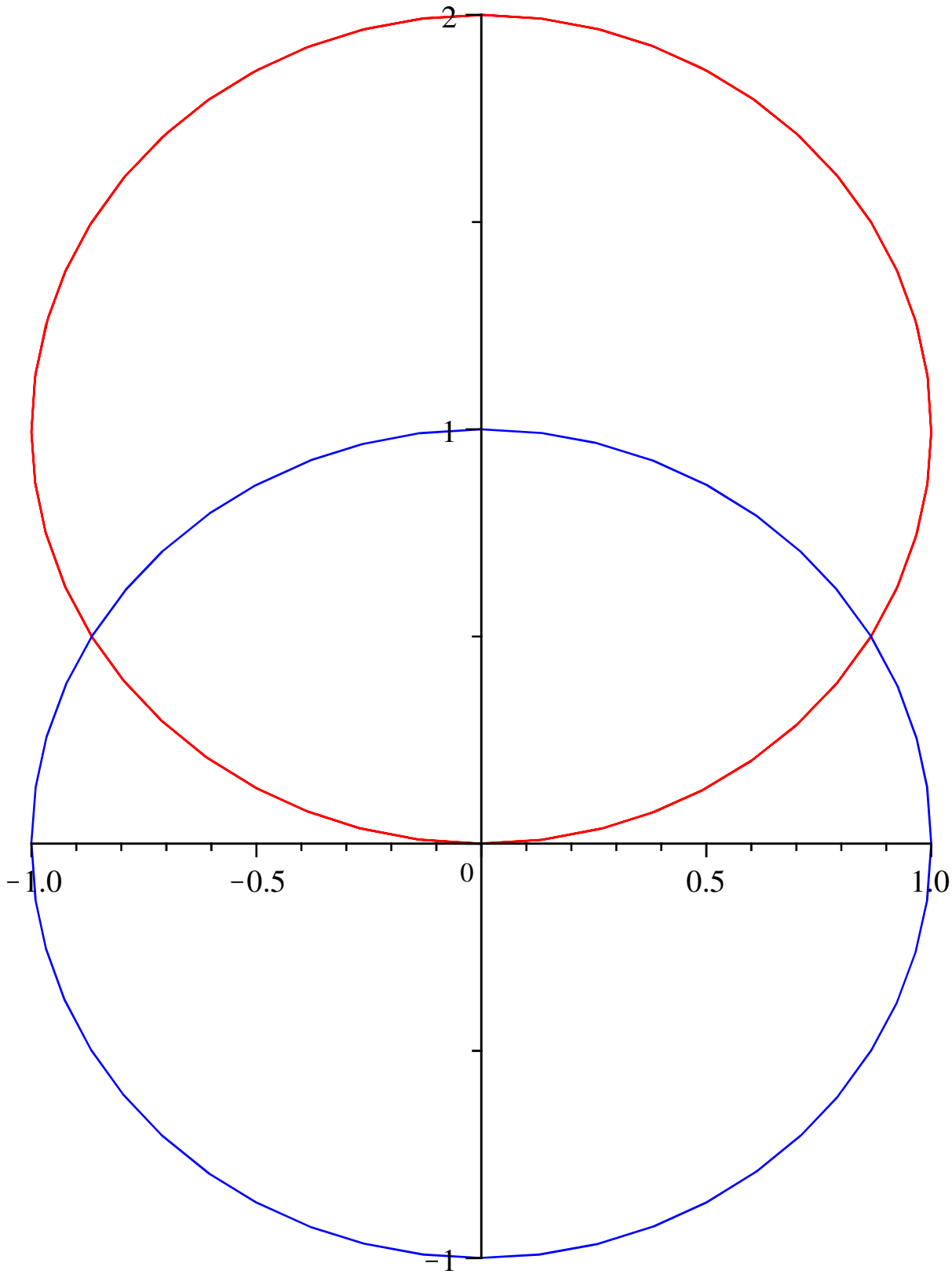
$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \left(2+2\sin(\theta)\right)^2 d\theta = 6\pi \qquad (3)$$

Ex4

```
> r1:= theta  ->  2*sin( theta); R1 := plot([r1(theta),theta,
  theta= 0 ..2*Pi],coords=polar,color=red):
  r2:= theta  ->  1; R2 := plot([r2(theta),theta,theta= 0 ..2*Pi],
  coords=polar,color=blue):
```

$$\theta \to 2\sin(\theta)$$
$$\theta \to 1 \qquad (4)$$

```
> display(R1,R2);
```



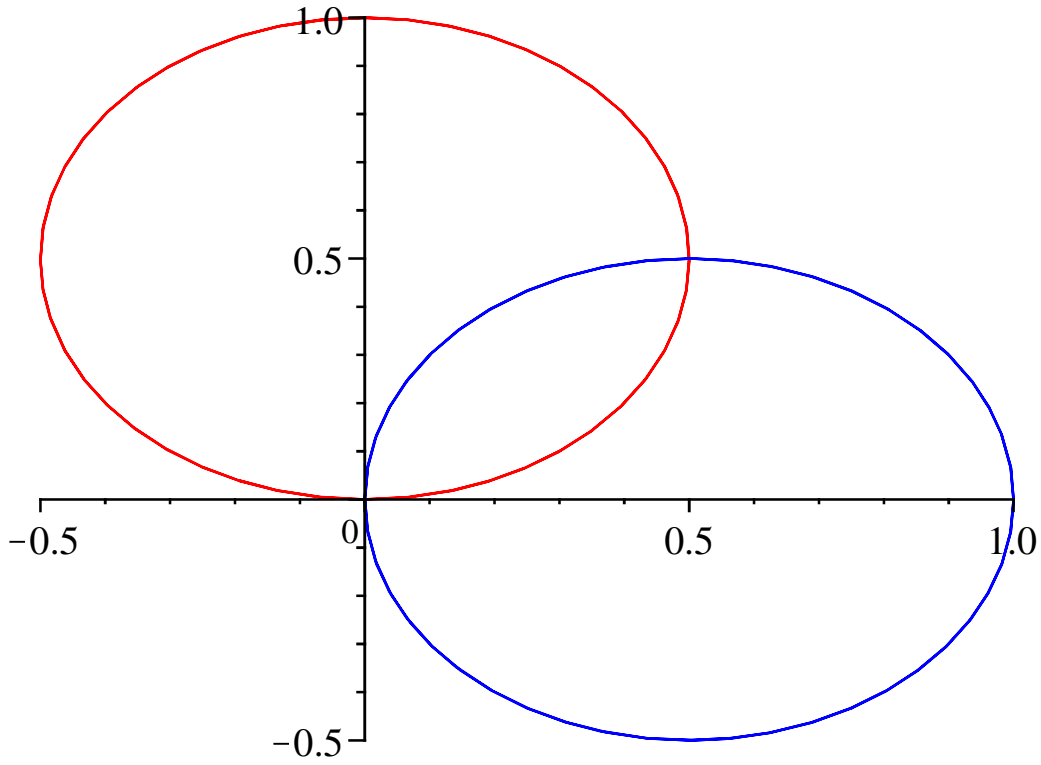

```
> 2*1/2*Int(r1^2 - r2^2,theta=Pi/6 .. Pi/2) = 2*1/2*int((r1(theta)
  )^2 - (r2(theta))^2,theta=Pi/6 .. Pi/2) ;
```

```
> p1:=plot([sin(t),t,t=0..2*Pi],coords=polar,color=red):
```

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \left(rI^2 - r2^2\right)\, d\theta = \frac{1}{2}\sqrt{3} + \frac{1}{3}\pi \qquad \textbf{(5)}$$

Ex5

```
> p2:=plot([cos(t),t,t=0..2*Pi],coords=polar,color=blue):
> p1:=plot([sin(t),t,t=0..2*Pi],coords=polar,color=red):
> display(p1,p2);
```

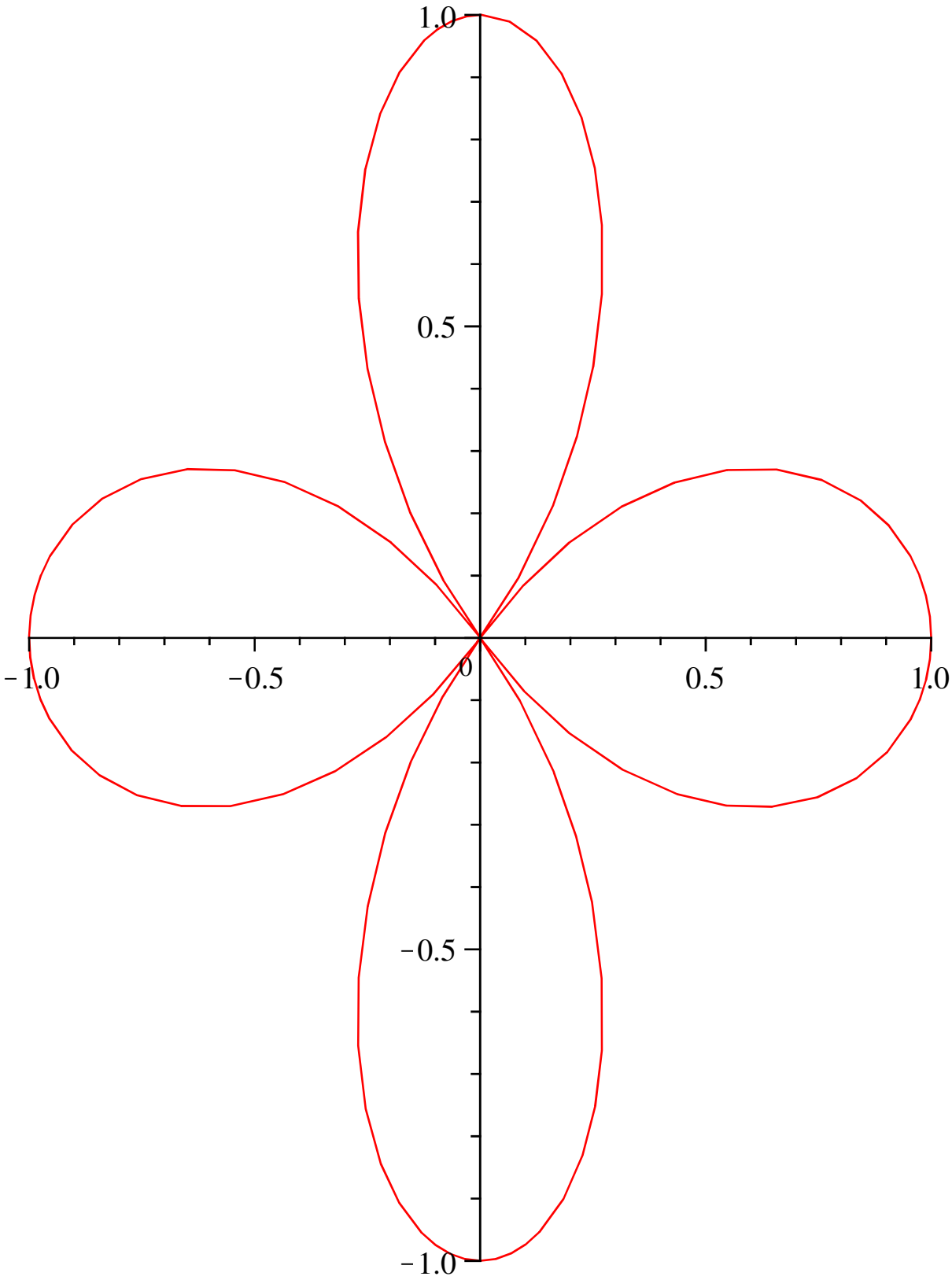



```
> 2*Int(1/2*(sin(theta))^2,theta=0 .. Pi/4)= 2*1/2*int((sin(theta)
  )^2,theta=0 .. Pi/4);
```

$$2\left(\int_0^{\frac{1}{4}\pi} \frac{1}{2}\sin(\theta)^2\, d\theta\right) = -\frac{1}{4} + \frac{1}{8}\pi \qquad \textbf{(6)}$$

Ex6

```
> plot([cos(2*theta),theta,theta=0..2*Pi],coords=polar,color=red);
```



>

note there are 4 leafs

```
> 8* Int(1/2*(cos(2*theta))^2,theta= 0 .. Pi/4)=8* int(1/2*(cos(2*
  theta))^2,theta= 0 .. Pi/4);
```

$$8\left(\int_0^{\frac{1}{4}\pi} \frac{1}{2}\cos(2\theta)^2\,d\theta\right) = \frac{1}{2}\pi \qquad (7)$$

```
> ?
```