```
restart:with (student): #solutions test3 m 1572 fall 2008

# question 1a
a(n) := ((-1)^n)* (ln(n)/n);
```

$$a(n) := \frac{(K1)^n \ln(n)}{n} \tag{1}$$

```
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} \left(\frac{(K1)^n \ln(n)}{n} \right) = 0 \tag{2}$$

```
# question 1b
a(n) :=  ( n - 1)/n: Limit(a(n), n=infinity)=limit(a(n), n =
infinity);
```

$$\lim_{n \to \infty} \left(\frac{n K1}{n} \right) = 1 \tag{3}$$

```
# 1c
a(n) :=  (1+2^n)/( 3^n):
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} \left(\frac{1 C 2^n}{3^n} \right) = 0 \tag{4}$$

```
# 1d
a(n) :=  ( 2 ^(2*n)/ 3^n):
Limit(a(n), n=infinity)=limit(a(n), n = infinity);
```

$$\lim_{n \to \infty} \left(\frac{2^{2n}}{3^n} \right) = N \tag{5}$$

```
# 2a
Sum(' (-1)^n*ln(n)/(n)', 'n'=1..N);
```

$$\sum_{n=1}^{N} \frac{(K1)^n \ln(n)}{n} \tag{6}$$

try integral test for the postive part (absolute values)

```
g:= x -> ln( x)/x:
Limit(Int(g(x),x=1 ..X),X=infinity)= limit(int(g(x),x=1 ..X),X=
infinity);
```

$$\lim_{X \to \infty} \left(\int_1^X \frac{\ln(x)}{x} \, dx \right) = N \tag{7}$$

so the

```
Limit(a(n), n=infinity)=limit(a(n), n = infinity); # nth term
```

goes to zero

$$\lim_{n \to N} \left(\frac{2^{2n}}{3^n} \right) = N \tag{8}$$

```
diff(g(x),x); # which is negative for x >= e
```

$$\frac{1}{x^2} \, K \, \frac{\ln(x)}{x} \tag{9}$$

```
Sum(' ln(n)/(n)', 'n'=1..N);# yields conditional convergence
```

$$\sum_{n=1}^{N} \frac{\ln(n)}{n} \tag{10}$$

2b

```
a(n) := (2^n)/(n!); Limit(a(n), n=infinity)=limit(a(n), n =
infinity);# check nth term go to zero
```

$$a(n) := \frac{2^n}{n!}$$

$$\lim_{n \to N} \left(\frac{2^n}{n!} \right) = 0 \tag{11}$$

```
#do a ratio test
```

```
b(n) := (2^(n+1))/((n+1)!);
```

$$b(n) := \frac{2^{n+1}}{(n+1)!} \tag{12}$$

```
Limit(b(n)/a(n),n=infinity)= limit(b(n)/a(n),n=infinity);
```

$$\lim_{n \to N} \left(\frac{2^{n+1} \, n!}{(n+1)! \, 2^n} \right) = 0 \tag{13}$$

```
# so it converges absolutely ratio test
```

2c

```
a(n) := (2)/(4*n^2-1); Limit(a(n), n=infinity)=limit(a(n), n =
infinity);# check nth term go to zero
```

$$a(n) := \frac{2}{4 n^2 - 1}$$

$$\lim_{n \to N} \left(\frac{2}{4 n^2 - 1} \right) = 0 \tag{14}$$

```
#do a integral test or comparison with 1/n^2
```

```
g:= x -> 2/(4*x^2-1):
Limit(Int(g(x),x=1 ..X),X=infinity)= limit(int(g(x),x=1 ..X),X=
infinity);
```

```
Warning, unable to determine if 1/2 is between 1 and X; try to use
assumptions or set _EnvAllSolutions to true
Warning, unable to determine if -1/2 is between 1 and X; try to use
```

assumptions or set _EnvAllSolutions to true

$$\lim_{x \to \infty} \left(\int_1^x \frac{2}{4x^2 + 1}\, dx \right) = \frac{1}{2} \ln(3) \tag{15}$$

converges absolutely bby integral test

a(n) := ((-1)^n) * n/(3^n); Limit(a(n), n=infinity)=limit(a(n), n = infinity);# check nth term go to zero

$$a(n) := \frac{(-1)^n n}{3^n}$$

$$\lim_{n \to \infty} \left(\frac{(-1)^n n}{3^n} \right) = 0 \tag{16}$$

#do a ratio test

b(n) := (n+1)/(3^(n+1));

$$b(n) := \frac{n + 1}{3^{n+1}} \tag{17}$$

Limit(b(n)/abs(a(n)),n=infinity)= limit(b(n)/abs(a(n)),n= infinity);

$$\lim_{n \to \infty} \left(\frac{n + 1}{3^{n+1} \left| \frac{(-1)^n n}{3^n} \right|} \right) = \frac{1}{3} \tag{18}$$

 # so it converges absolutely by ratio test

?

"?" (19)

a(n) := ((-1)^n) * (2*n+1)/(5*n+7); Limit(a(n), n=infinity)= limit(a(n), n = infinity);# check nth term go to zero

$$a(n) := \frac{(-1)^n (2n + 1)}{5n + 7}$$

$$\lim_{n \to \infty} \left(\frac{(-1)^n (2n + 1)}{5n + 7} \right) = -\frac{2}{5} \quad -\frac{2}{5} \tag{20}$$

so it diverges by the nth term test

?

"?" (21)

a(n) := (2^n)*(x^n)/(n^2);

$$a(n) := \frac{2^n x^n}{n^2} \tag{22}$$

#do a ratio test

b(n) := (2^(n+1))*x^(n+1)/((n+1)^2);

$$b(n) := \frac{2^{n+1}\,x^{n+1}}{(n+1)!} \tag{23}$$

```
Limit(b(n)/a(n),n=infinity)= limit(b(n)/a(n),n=infinity);
```

$$\lim_{n\to\infty}\left(\frac{2^{n+1}\,x^{n+1}\,n^2}{(n+1)!\,2^n\,x^n}\right) = 2\,x \tag{24}$$

```
# so it converges absolutely ratio test for !x!!   (1/2)  check ends
```

```
a(n) := ((-1)^n)*(x^n)/(n^2);
```

$$a(n) := \frac{(-1)^n\,x^n}{n^2} \tag{25}$$

```
#do a ratio test
```

```
b(n) := x^(n+1)/((n+1)^2);a(n) := (x^n)/(n^2);
```

$$b(n) := \frac{x^{n+1}}{(n+1)^2}$$

$$a(n) := \frac{x^n}{n^2} \tag{26}$$

```
Limit(b(n)/(a(n)),n=infinity)= limit(b(n)/(a(n)),n=infinity);
```

$$\lim_{n\to\infty}\left(\frac{x^{n+1}\,n^2}{(n+1)^2\,x^n}\right) = x \tag{27}$$

```
# so it converges absolutely ratio test for !x!!   1  check ends
```

Error, missing denominator

$$\frac{\text{so it converges absolutely ratio test for } !x!!\quad 1\quad \text{check ends}}{0}$$