Name _____

Final Exam Math. 2673 Fall, 2006

I.)

1.) Graph the region and find the volume inside both the sphere $x^2 + y^2 + z^2 = 4$ and exterior to the sphere $x^2 + y^2 + z^2 = 1$.

2.) Graph the region and find the volume bounded by the cone whose angle in spherical coordinates is given by $\phi = \frac{\pi}{3}$ and a sphere of radius 2.

II

3.) Verify Green's Theorem for $\int_C (x - y)dx + (x + y)dy$ where C is in the counterclockwise direction about the triangle with vertices at $(0, 0), (0, 1), (1, 0),)$

4.) Integrate $f(x, y) = x + y$ over the curve formed by C_1 and C_2

C_1 :line seqment between $(0, 0) to (1, 1)$ and

C_2 : line seqment between $(1, 1) to (2, 3)$

tem5.) Let $F(x, y) = y\imath + x\jmath$ be a continuous velocity field and consider the closed path γ given by the $r(t)$ which is the union of the arch $r_1(t) = \cos(t)\imath + \sin(t)\jmath, 0 \leq t \leq \pi$

followed by the line segment ($r_2(t)$) $(-1, 0)$, and $(1, 0)$

Find the $\int_\gamma F(x, y)\dot{dr}$

III.)

6.) Find $f(x, y, z)$ such that $\nabla f = F$ for each of the following:

a.) $F(x, y, z) = (yz, xz, xy)$

7.) For $F(x, y, z) = \sin(x)\imath + \sin(y)\jmath + xz^2k$

a.) Find $\nabla(\nabla \cdot F)$

b.) Find $\nabla \times (\nabla \times F)$

IV.)

8.)

a.) Graph the region of integration and evaluate:

$\int_0^3 \int_{x-2}^{(x)} (x+y) dy dx.$

b.) Using the change of variable $x = u + v$ and $y = v$ graph the new region, evaluate the Jacobian and evaluate the integral.