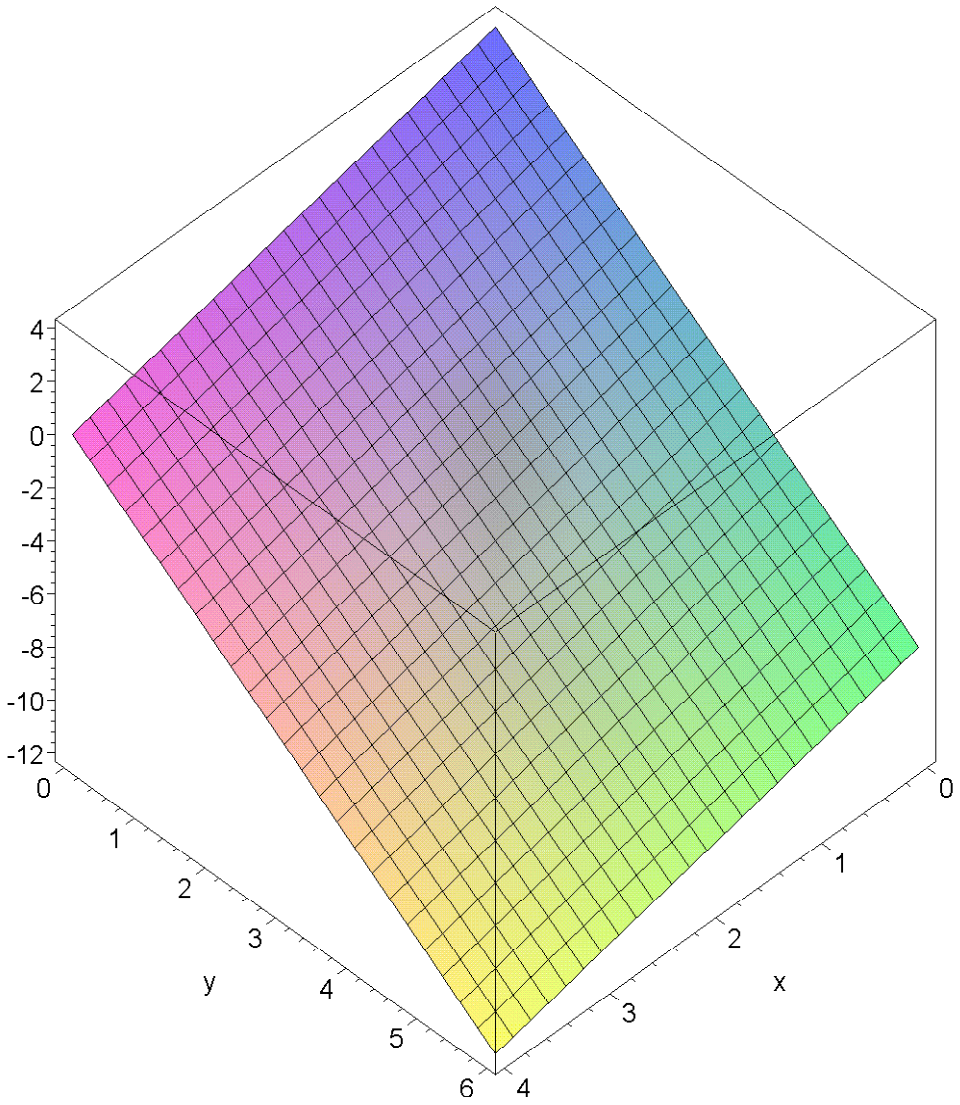
```
> restart:with(student):
  with(plots):# solutions to exam 3
Warning, the name changecoords has been redefined
```

```
> #1
```

```
>  plot3d(4-x-2*y,x=0..4, y=0 .. 6,style=patch,axes = boxed);
```

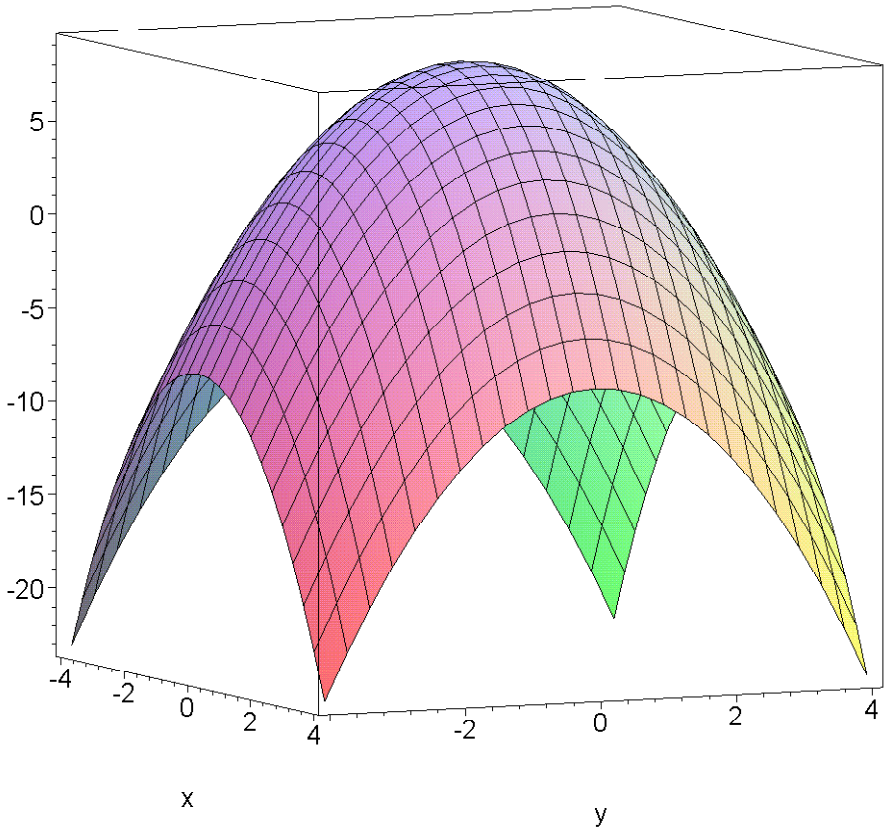



```
> Int(Int(Int(1, z=0..4-x-2*y),y=0 ..(4-x)/2), x=0..4)
  = int(int(int(1, z=0..4-x-2*y),y=0 ..(4-x)/2), x=0..4);
```

$$\int_0^4 \int_0^{2-\frac{x}{2}} \int_0^{4-x-2y} 1 \, dz \, dy \, dx = \frac{16}{3}$$

```
> #2
```

```
>  plot3d(9-x^2-y^2,x=-4..4, y=-4 .. 4,style=patch,axes = boxed);
```



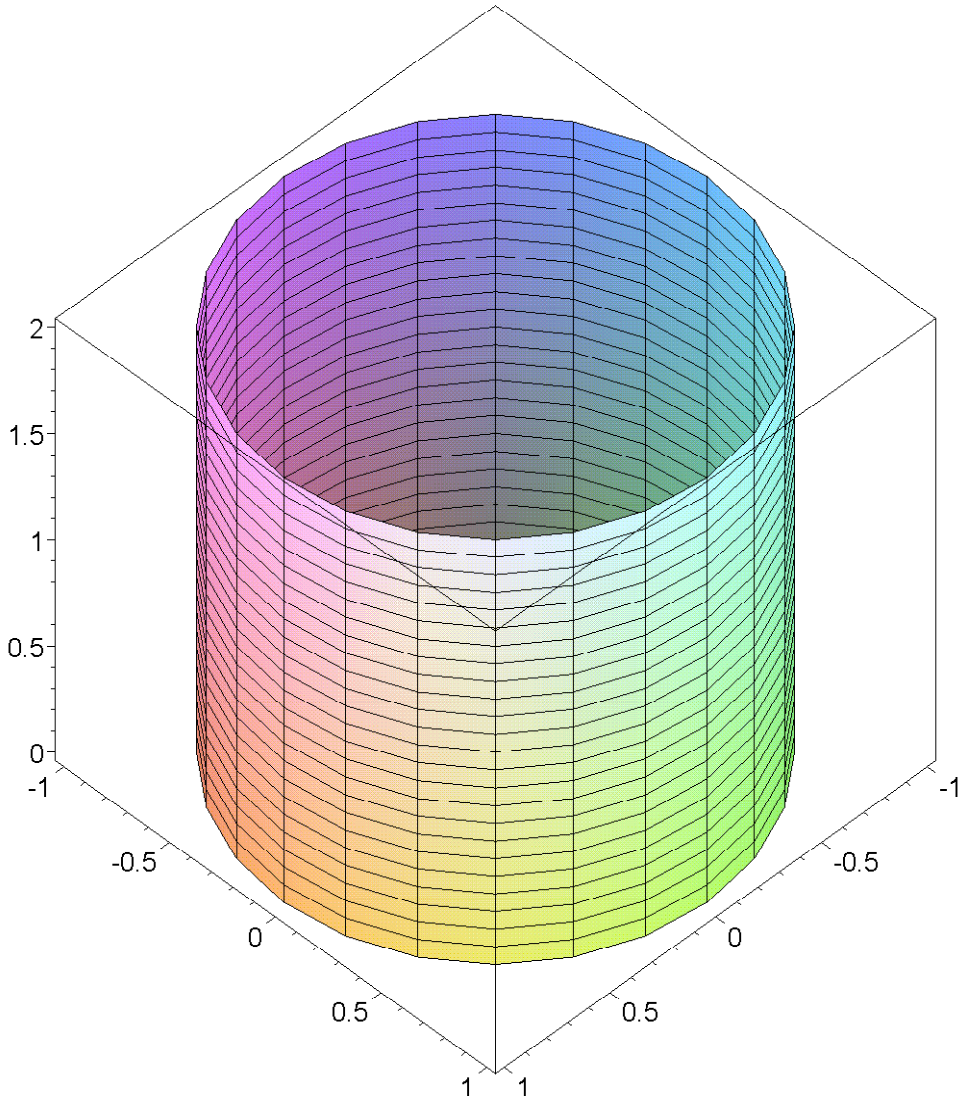

```
> Int(Int(Int(1, z=0..sqrt(9-x^2-y^2)),y=0 ..sqrt(9-x^2)), x=0..3)
  = int(int(int(1, z=0..sqrt(9-x^2-y^2)),y=0 ..sqrt(9-x^2)), x=0..3);
  # or
```

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} 1 \, dz \, dy \, dx = \frac{9\,\pi}{2}$$

```
> Int(Int(Int(rho^2* sin(phi), rho=0..3),phi=0 ..Pi/2), theta=0..Pi/2)
  = int(int(int(rho^2* sin(phi), rho=0..3),phi=0 ..Pi/2), theta=0..Pi/2);
```

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^3 \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = \frac{9\,\pi}{2}$$

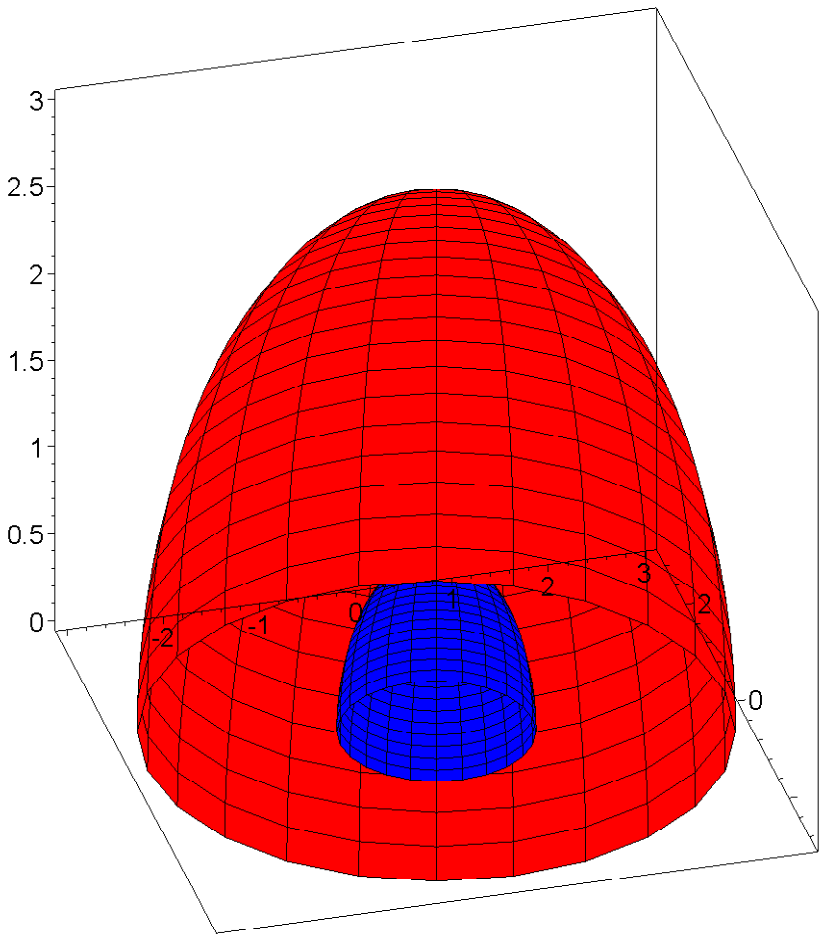
```
> #3
>  plot3d(1, theta= 0 .. 2*Pi,z=0..2,coords=cylindrical,style=patch,axes = boxed);
```




```
> Int(Int(Int(r, z=0..2),r=0 ..1), theta=0..Pi*2)
  = int(int(int(r, z=0..2),r=0 ..1), theta=0..Pi*2);
```

$$\int_0^{2\pi}\int_0^1\int_0^2 r\,dz\,dr\,d\theta = 2\,\pi$$

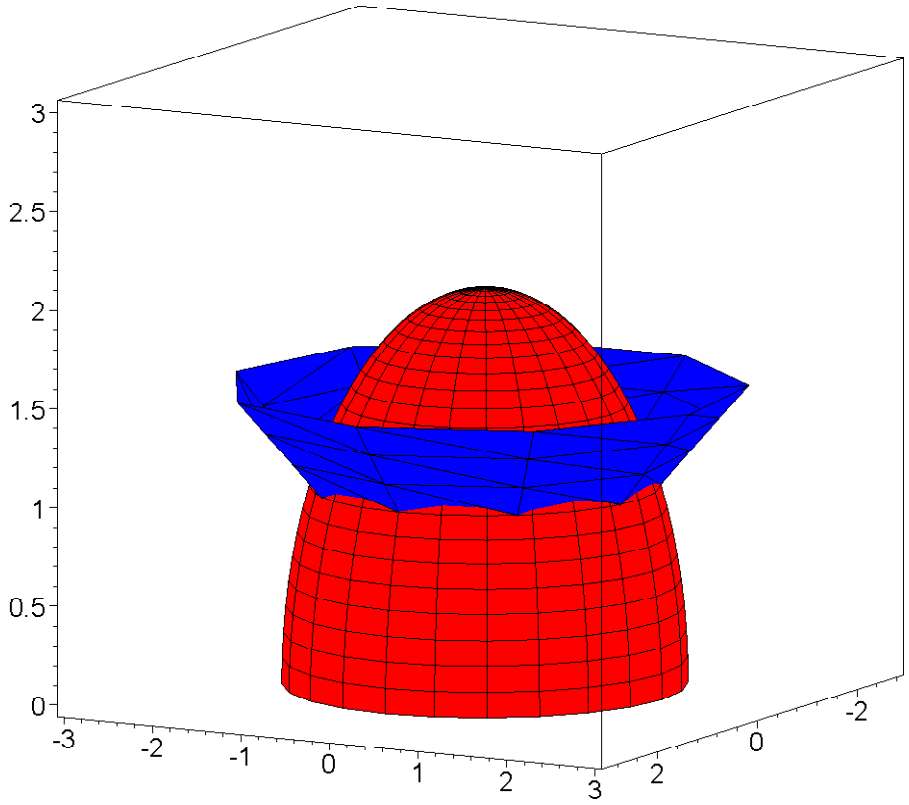
```
> #4
>  P1:=plot3d(3, theta= 0 .. 2*Pi,phi=0..Pi/2,coords=spherical,color =
  red,style=patch,axes = boxed):
  P2:=plot3d(1, theta= 0 .. 2*Pi,phi=0..Pi/2,coords=spherical,color =
  blue,style=patch,axes = boxed):

> display3d(P1,P2);
```




```
> Int(Int(Int(rho^2*sin(phi), rho=1..3),phi=0 ..Pi), theta=0..Pi*2)
  = int(int(int(rho^2*sin(phi), rho=1..3),phi=0 ..Pi), theta=0..Pi*2) ;
```

$$\int_0^{2\pi}\int_0^{\pi}\int_1^{3} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{104}{3}\pi$$

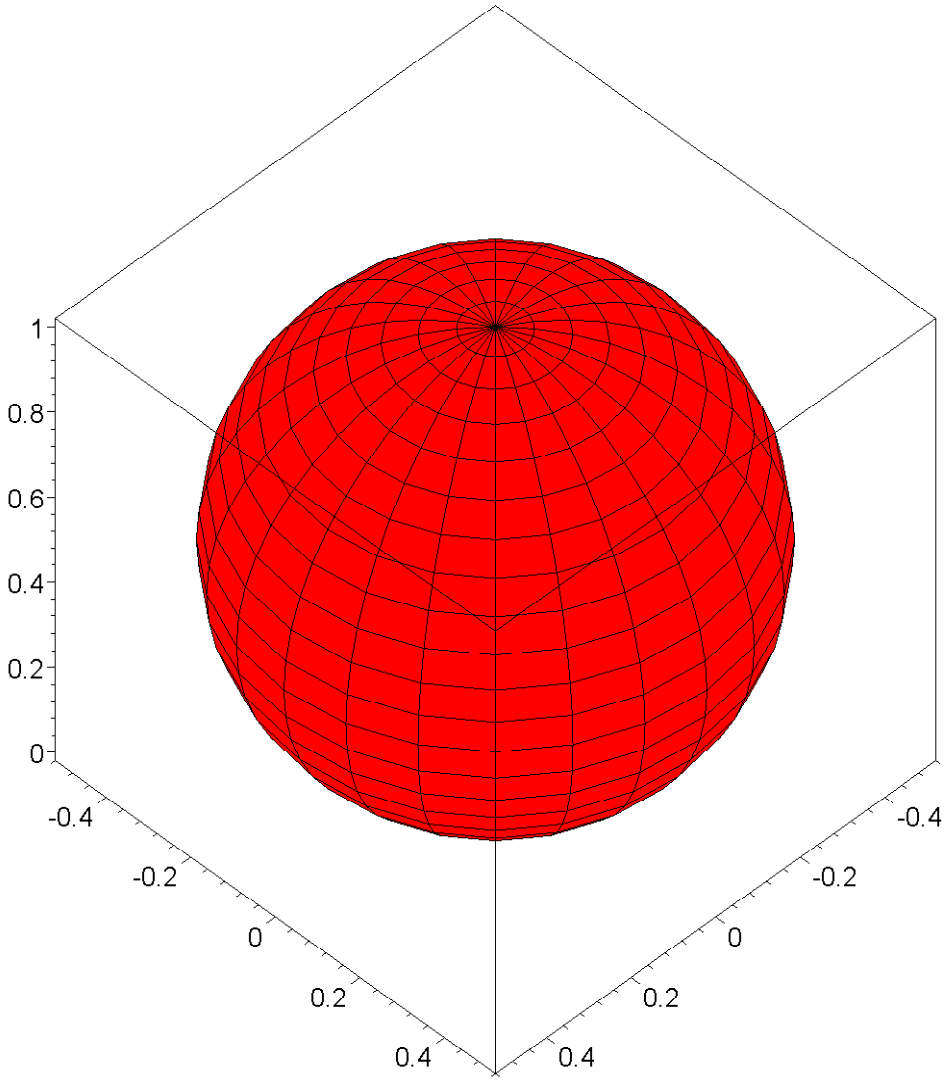
```
> 
> #5
>  P1:=plot3d(2, theta= 0 .. 2*Pi,phi=0..Pi/2,coords=spherical,color =
  red,style=patch,axes = boxed):
  P2:=implicitplot3d(phi = Pi/3,rho=0..3, theta= 0 ..
  2*Pi,phi=0..Pi/2,coords=spherical,color = blue,style=patch,axes = boxed):
> display3d(P1,P2);
```




```
> Int(Int(Int(rho^2*sin(phi), rho=0..2),phi=0 ..Pi/3), theta=0..Pi*2)
  = int(int(int(rho^2*sin(phi), rho=0..2),phi=0 ..Pi/3), theta=0..Pi*2) ;
```

$$\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{2}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta = \frac{8\pi}{3}$$

```
> #6
>   plot3d(cos(phi), theta= 0 .. 2*Pi,phi=0..Pi/2,coords=spherical,color =
  red,style=patch,axes = boxed);
```




```
>
```

```
> Int(Int(Int(rho^2*sin(phi), rho=0..cos(phi)),phi=0 ..Pi/2), theta=0..Pi*2)
  = int(int(int(rho^2*sin(phi), rho=0..cos(phi)),phi=0 ..Pi/2), theta=0..Pi*2) ;
```

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{\cos(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta = \frac{\pi}{6}$$

```
>
>
```