```
> restart:
> with (student):with(plots):
  with(linalg):
> f:=(x,y)-> ln( e^x + e^y);
```

$$f := (x, y) \to \ln\left(e^x + e^y\right) \tag{1}$$

```
> Diff(f(x,y),x) +Diff(f(x,y),y)=diff(f(x,y),x) +diff(f(x,y),y);
```

$$\frac{\partial}{\partial x}\ln\left(e^x + e^y\right) + \frac{\partial}{\partial y}\ln\left(e^x + e^y\right) = \frac{e^x \ln(e)}{e^x + e^y} + \frac{e^y \ln(e)}{e^x + e^y} \tag{2}$$

```
> simplify(%);
```

$$\frac{\partial}{\partial x}\ln\left(e^x + e^y\right) + \frac{\partial}{\partial y}\ln\left(e^x + e^y\right) = \ln(e) \tag{3}$$

ex #2

```
> restart:with (student):
> f:=(x,y)-> 3*x*y-x^2*y-x*y^2;
```

$$f := (x, y) \to 3\,y\,x - x^2\,y - x\,y^2 \tag{4}$$

```
> Diff(f(x,y),x)=diff(f(x,y),x);Diff(f(x,y),y)=diff(f(x,y),y);
```

$$\frac{\partial}{\partial x}\left(3\,y\,x - x^2\,y - x\,y^2\right) = 3\,y - 2\,y\,x - y^2$$

$$\frac{\partial}{\partial y}\left(3\,y\,x - x^2\,y - x\,y^2\right) = 3\,x - x^2 - 2\,y\,x \tag{5}$$

```
> fx:=(x,y)->diff(f(x,y),x);fy:=(x,y)->diff(f(x,y),y);
```

$$fx := (x, y) \to \frac{\partial}{\partial x} f(x, y)$$

$$fy := (x, y) \to \frac{\partial}{\partial y} f(x, y) \tag{6}$$

```
> fxy:=diff(fx(x,y),y);fxx:=diff(fx(x,y),x);fyy:=diff(fy(x,y),y);
```

$$fxy := 3 - 2\,x - 2\,y$$

$$fxx := -2\,y$$

$$fyy := -2\,x \tag{7}$$

```
> xf:=diff(f(x,y),x);yf:=diff(f(x,y),y); solve({xf=0,yf=0},{x,y});
```

$$xf := 3\,y - 2\,y\,x - y^2$$

$$yf := 3\,x - x^2 - 2\,y\,x$$

$$\{x = 0, y = 0\},\ \{x = 3, y = 0\},\ \{x = 0, y = 3\},\ \{x = 1, y = 1\} \tag{8}$$

```
> disc:= (x,y)-> diff(f(x,y),x,x)*diff(f(x,y),y,y)-diff(f(x,y),x,
  y)*diff(f(x,y),x,y);DXY:=fxx*fyy-fxy*fxy;
```

$$disc := (x, y) \to \left(\frac{\partial^2}{\partial x^2} f(x, y)\right)\left(\frac{\partial^2}{\partial y^2} f(x, y)\right) - \left(\frac{\partial^2}{\partial y\,\partial x} f(x, y)\right)\left(\frac{\partial^2}{\partial y\,\partial x} f(x, y)\right)$$

$$DXY := 4\,y\,x - (3 - 2\,x - 2\,y)^2 \tag{9}$$

```
> subs(x=0,y=0,DXY);
```

$$-9 \tag{10}$$

```
> subs(x=3,y=0,DXY);
```

$$-9 \tag{11}$$

```
> subs(x=0,y=3,DXY);
```

$$-9 \tag{12}$$

```
> subs(x=1,y=1,DXY );
> subs(x=1,y=1,fxx);#local max
```

$$\begin{matrix} 3 \\ -2 \end{matrix} \tag{13}$$

ex 3

```
> restart:with (student):z:= (x,y)->cos(y)+(x);
```

$$z := (x, y) \rightarrow \cos(y) + x \tag{14}$$

```
> X:= (u,v)-> u^2+v;Y:= (u,v)-> u-v^2;
```

$$X := (u, v) \rightarrow u^2 + v$$
$$Y := (u, v) \rightarrow u - v^2 \tag{15}$$

```
> Diff(z(x,y),x)*Diff(X(u,v),u)+Diff(z(x,y),y)*Diff(Y(u,v),u)=
  diff(z(x,y),x)*diff(X(u,v),u)+diff(z(x,y),y)*diff(Y(u,v),u);
```

$$\left(\frac{\partial}{\partial x}(\cos(y) + x)\right)\left(\frac{\partial}{\partial u}(u^2 + v)\right) + \left(\frac{\partial}{\partial y}(\cos(y) + x)\right)\left(\frac{\partial}{\partial u}(u - v^2)\right) = 2\,u - \sin(y) \tag{16}$$

```
> subs(x=u^2+v,y=u-v^2,%);simplify(%);
```

$$(Diff(\cos(u - v^2) + u^2 + v, u^2 + v))\left(\frac{\partial}{\partial u}(u^2 + v)\right) + (Diff(\cos(u - v^2) + u^2 + v, u - v^2))\left(\frac{\partial}{\partial u}(u - v^2)\right) = 2\,u - \sin(u - v^2)$$
$$(Diff(\cos(u - v^2) + u^2 + v, u^2 + v))\left(\frac{\partial}{\partial u}(u^2 + v)\right) + (Diff(\cos(u - v^2) + u^2 + v, u - v^2))\left(\frac{\partial}{\partial u}(u - v^2)\right) = 2\,u - \sin(u - v^2) \tag{17}$$

```
> Diff(z(x,y),x)*Diff(X(u,v),v)+Diff(z(x,y),y)*Diff(Y(u,v),v)=
  diff(z(x,y),x)*diff(X(u,v),v)+diff(z(x,y),y)*diff(Y(u,v),v);
```

$$\left(\frac{\partial}{\partial x}(\cos(y) + x)\right)\left(\frac{\partial}{\partial v}(u^2 + v)\right) + \left(\frac{\partial}{\partial y}(\cos(y) + x)\right)\left(\frac{\partial}{\partial v}(u - v^2)\right) = 1 + 2\sin(y)\,v \tag{18}$$

```
> subs(x=u^2+v,y=u-v^2,%);simplify(%);
```

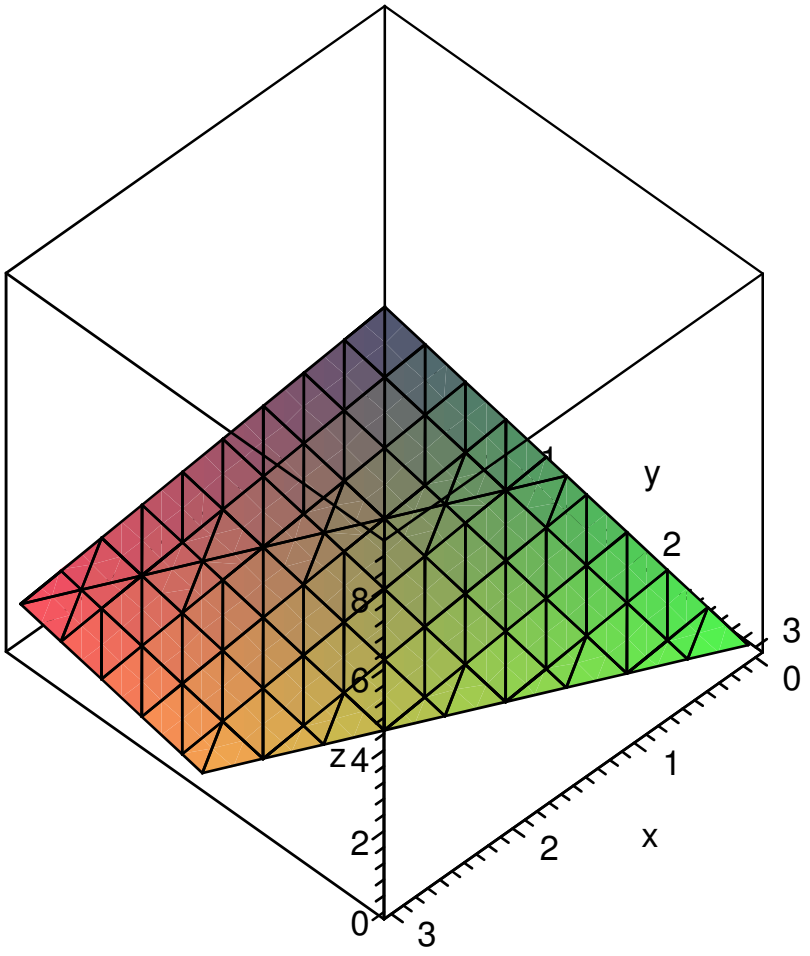
$$(Diff(\cos(u - v^2) + u^2 + v, u^2 + v))\left(\frac{\partial}{\partial v}(u^2 + v)\right) + (Diff(\cos(u - v^2) + u^2 + v, u - v^2))\left(\frac{\partial}{\partial v}(u - v^2)\right) = 1 + 2\sin(u - v^2)\,v$$
$$(Diff(\cos(u - v^2) + u^2 + v, u^2 + v))\left(\frac{\partial}{\partial v}(u^2 + v)\right) + (Diff(\cos(u - v^2) + u^2 + v, u - v^2))\left(\frac{\partial}{\partial v}(u - v^2)\right) = 1 + 2\sin(u - v^2)\,v \tag{19}$$

PART 2

```
> restart:with (student):with(linalg):
```

ex1

```
> restart:with (student):with(linalg):with(plots):
> implicitplot3d(x +2*y +3*z =6,x=0..3,y= 0..3,z=0..9,axes = box);
```




```
>
> Int(Int( (6 − x − 3·y)/3 , x = 0..6 - 2·y), y = 0..3);
```

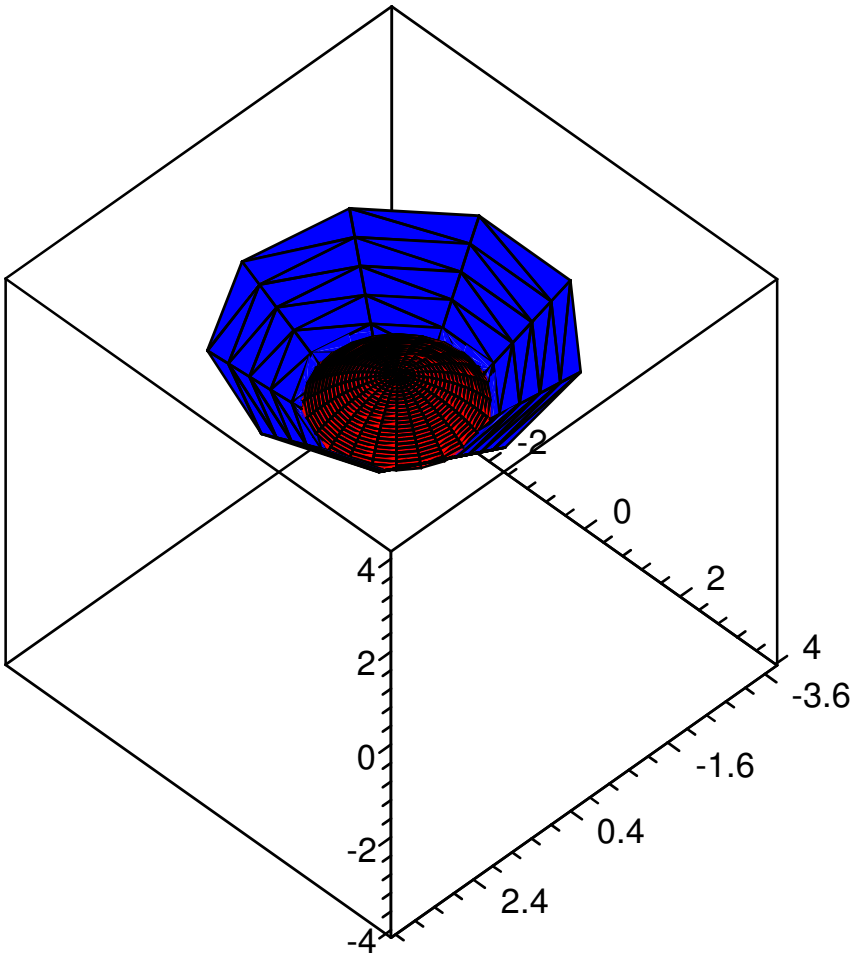
$$\int_0^3 \int_0^{6-2y} \left(2 - \frac{1}{3}x - y\right) dx\, dy \qquad (20)$$

```
> int(int( (6 − x − 3·y)/3 , x = 0..6 - 2·y), y = 0..3);
```

$$3 \qquad (21)$$

```
> plot1:=plot3d(2, theta=0..2*Pi, phi=0 .. Pi/4, coords=spherical,
  color = red, axes=boxed):
> plot2:=implicitplot3d(phi=Pi/4,rho=0..4,theta=0..2*Pi,phi=0..Pi,
  coords=spherical , color = blue,axes = boxed):display3d(plot1,
```

```
plot2);
Int(Int(Int(rho^2*sin(phi), rho=0..2), phi=0..Pi/4),theta=0..Pi*
2)= int(int(int(rho^2*sin(phi), rho=0..2), phi=0..Pi/4),theta=0.
.Pi*2);
>
```



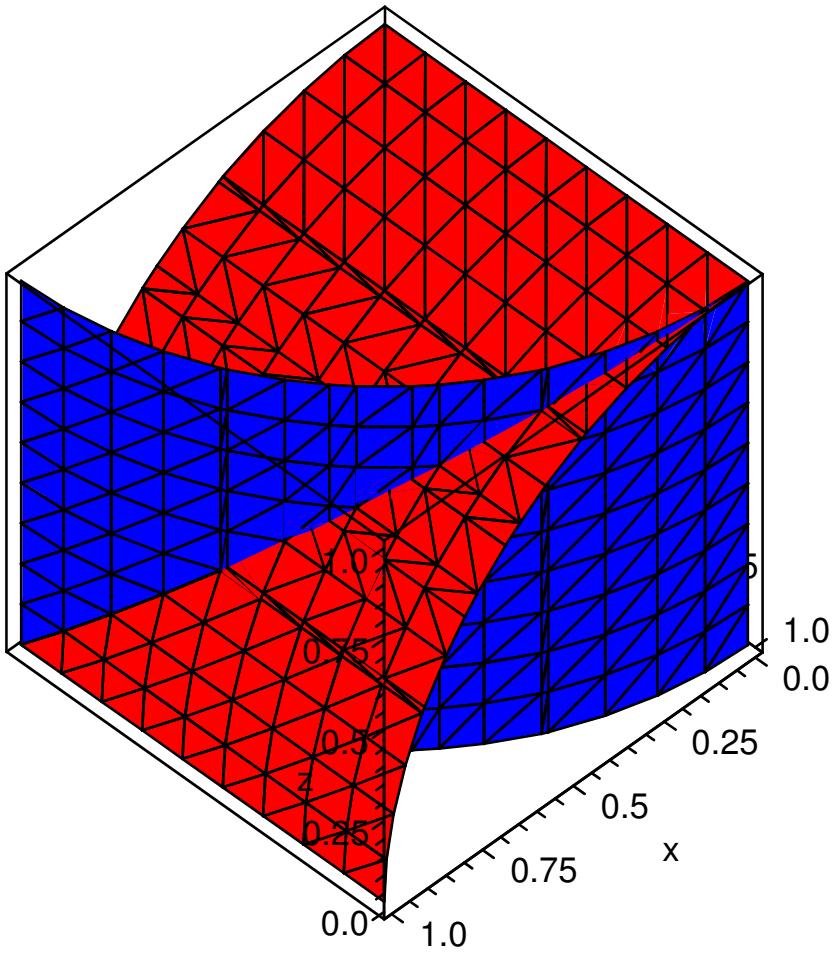
$$\int_0^{2\pi}\int_0^{\frac{1}{4}\pi}\int_0^{2}\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=\frac{16}{3}\pi-\frac{8}{3}\sqrt{2}\,\pi \qquad (22)$$

> $Int(Int(3, x=0..1), y=0..2)=int(int(3, x=0..1), y=0..2);$

$$\int_0^{2}\int_0^{1}3\,dx\,dy=6 \qquad (23)$$

> $restart: with(student): with(linalg): with(plots): plot2:=implicitplot3d(x^2+z^2=1, x=0..1, y=0..1, z=0..1, color=red, axes=boxed):$

```
> plot1:=implicitplot3d(x^2 + y^2 = 1,x=0..1,y = 0 ..1,z=0 .. 1,
color = blue,axes = boxed):display3d(plot1,plot2);
```




```
> restart : with (student) : with(linalg) : with(plots) :
> Int(Int( Int (r, z=0..sqrt(1 - (r*cos(theta))^2 )), r=0..1),
  theta=0..Pi/2);
>
```

$$\int_0^{\frac{1}{2}\pi}\int_0^1\int_0^{\sqrt{1-r^2\cos(\theta)^2}} r\,dz\,dr\,d\theta \tag{24}$$

```
>
>
> (pi/2 + 2/3)/3;
```

$$\frac{1}{6}\pi + \frac{2}{9} \tag{25}$$

```
> restart : with(plots) : with(student) : with(VectorCalculus) :
> SetCoordinates( 'cartesian'[x,y,z] );
```

$$cartesian_{x,\,y,\,z} \tag{26}$$

```
> v := VectorField( <y*z,x*z,x*y+2*z> );
```

$$v := (y\,z)\bar{e}_x + (x\,z)\bar{e}_y + (x\,y + 2\,z)\bar{e}_z \tag{27}$$

```
> ScalarPotential( v );
```

$$y\,z\,x + z^2 \tag{28}$$

```
> restart : with(plots) : with(student) : with(VectorCalculus) : SetCoordinates( 'cartesian'[x,
    y] );
```

$$cartesian_{x,\,y} \tag{29}$$

```
> LineInt( VectorField( <3*x^2*y,x^3+2*y> ), Path( <cos(t),sin(t)
  >, t=0..Pi ),inert )+ LineInt( VectorField( <3*x^2*y,x^3+2*y> ),
  LineSegments( <-1,0>, <1,0>),inert )= (LineInt( VectorField( <3*
  x^2*y,x^3+2*y> ), Path( <cos(t),sin(t)>, t=0..Pi ) )+ LineInt(
  VectorField( <3*x^2*y,x^3+2*y> ), LineSegments( <1,0>, <1,0>) ))
  ;
```

$$\int_0^{\pi} \left(-3\cos(t)^2 \sin(t)^2 + \left(\cos(t)^3 + 2\sin(t)\right)\cos(t)\right) dt + \int_0^{1} 0\,dt = 0 \tag{30}$$

```
> restart : with(plots) : with(student) : with(VectorCalculus) : SetCoordinates( 'cartesian'[x, y,
    z] );
```

$$cartesian_{x,\,y,\,z} \tag{31}$$

```
> F := VectorField( <2*x*y,z,cos(z)> );
```

$$F := 2\,x\,y\bar{e}_x + (z)\bar{e}_y + (\cos(z))\bar{e}_z \tag{32}$$

```
> Divergence( F );
```

$$2\,y - \sin(z) \tag{33}$$

```
> Gradient(%);
```

$$2\bar{e}_y - \cos(z)\bar{e}_z \tag{34}$$

```
> # b
```

```
> Curl( F );
```

$$-\bar{e}_x - 2\,x\bar{e}_z \tag{35}$$

```
> Divergence(%);
```

$$0 \tag{36}$$

```
>
```